AUTOMATIC LIE POISSON SYNTHESIS BASED ON GENETIC ALGORITHMS

SEAN BREWER
Department of Mathematics and Computer Science
Berry College, Mount Berry, Georgia USA

Abstract

Lie Poisson (LP) synthesis is presented as a generalization of FM synthesis. As the parameter space for LP synthesis is large, random searching for correct parameters to simulate various instruments such as piano, trumpet or voice is impractical. We present a method of using genetic algorithms implemented on the Berry Grid to search parameter spaces to model such instruments.

Introduction

A musical instrument is any device that can be used to produce musical tones or sounds. Common examples of musical instruments include the trumpet, saxophone, and guitar.

In order to simulate the sounds that are made by instruments, sound synthesis, which is an electronic or digital method to simulate sound, may be used. Common methods of sound synthesis include additive synthesis which is the combining of tones, usually harmonics of varying amplitudes, subtractive synthesis which is the filtering of complex sounds to shape harmonic spectrum, usually starting with geometric waves, and frequency modulation (FM) synthesis which is the modulating of a carrier wave with one or more operator. FM synthesis is capable of modelling any instrument, which means FM synthesis is capable of modelling any sound that is a constant tone or a tone with vibrato. However, FM synthesis cannot model tones which are chaotic in nature, such as wind or white noise. Lie Poisson synthesis is an improvement upon FM synthesis, since it can do everything that FM synthesis can do as well as having the capability to model chaotic tones.

It is possible to model any desired tone with Lie Poisson synthesis, however, it is very difficult to do. If parameters are randomly chosen it is highly unlikely that a tone will be a desirable or non-chaotic tone. To find parameters that model an instrument with LP synthesis in a short amount of time, genetic algorithms, or algorithms that emulate evolution and natural selection to solve a problem, have been implemented. These algorithms find the best set of parameters based on a recording of an instrument that model the given instrument.

Lie Poisson Equations

Equations:

$$
\begin{aligned}
x_1' &= y_1 z_1(\beta_1 - \gamma_1) - \epsilon y_2 z_1 + \zeta z_2 y_1 \\
y_1' &= z_1 x_1(\gamma_1 - \alpha_1) - \zeta z_2 x_1 + \delta x_2 z_1 \\
z_1' &= x_1 y_1(\alpha_1 - \beta_1) - \delta x_2 y_1 + \epsilon y_2 x_1 \\
x_2' &= y_2 z_2(\beta_2 - \gamma_2) - \epsilon y_1 z_2 + \zeta z_1 y_2 \\
y_2' &= z_2 x_2(\gamma_2 - \alpha_2) - \zeta z_1 x_2 + \delta x_1 z_2 \\
z_2' &= x_2 y_2(\alpha_2 - \beta_2) - \delta x_1 y_2 + \epsilon y_1 x_2
\end{aligned}
$$

Initial Conditions:

$$
\begin{aligned}
x_1(0) &= \cos(\theta)\sin(\phi) & x_2(0) &= 0 \\
y_1(0) &= \sin(\theta)\sin(\phi) & y_2(0) &= 1 \\
z_1(0) &= \cos(\phi) & z_2(0) &= 0
\end{aligned}
$$

Parameters:

$$
\begin{matrix}
\alpha_1 & \beta_1 & \gamma_1 & \delta & \epsilon & \zeta \\
\alpha_2 & \beta_2 & \gamma_2 & \theta & \phi &
\end{matrix}
$$

The set of equations above are a non-linear differential system in 6 variables with 9 parameters. The equations are solved with an advanced numerical algorithm where $x_1(t)$ is the output signal.

In the diagrams below, the light graph is the original signal and the dark graph is the signal generated with the algorithm.

Genetic Algorithm Theory

The genetic algorithm that is used by the program to find the best set of parameters is very straightforward. As stated earlier, a genetic algorithm simulates natural selection and evolution in order to solve a problem. The basic process in which the algorithm works is very simple:

1. Choose the initial population of parameters.

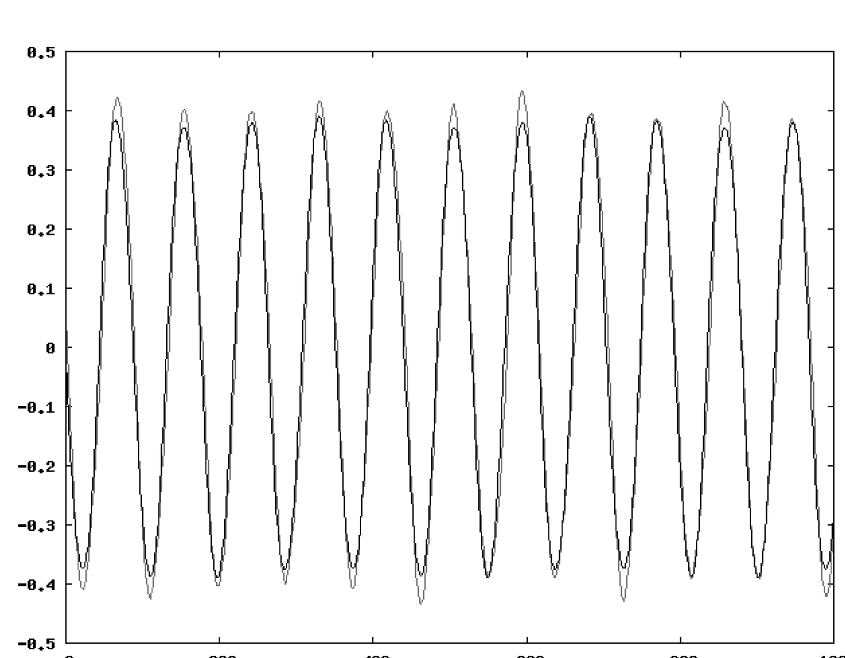
2. Evaluate the fitness of each individual parameter set in the population by comparing the output of the algorithm to the desired signal.

3. Repeat:
    1. Select the best ranking individuals to reproduce
    2. Breed new generation through crossover and mutation and give birth to offspring.
    3. Evaluate the individual fitnesses of the offspring.
    4. Replace the worst ranked part of the population with the offspring.

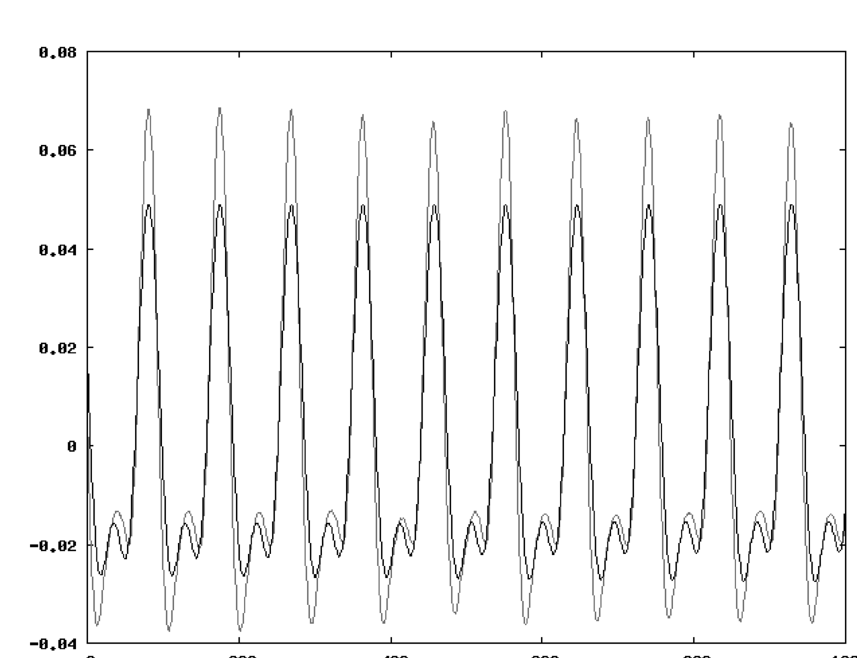
4. Terminate.

Results and Conclusion

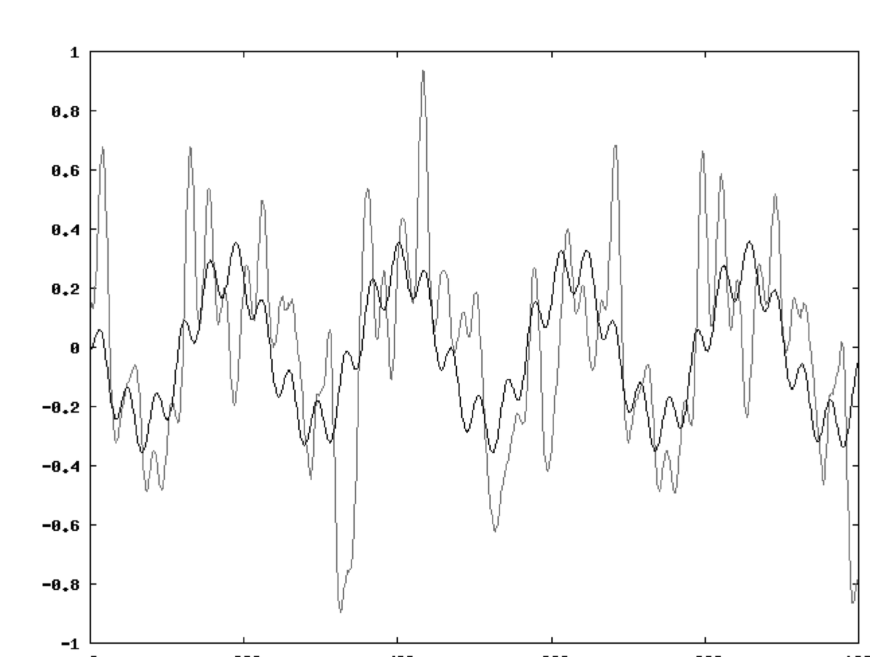
We were able to generate signals that were surprisingly close to the referenced instrument tones although there were differences in the high-frequency components in the spectrum. This is very exciting new research that definitely can be improved upon to hopefully model much more complex instruments evolving in time.



Wooden pipe



Flugelhorn



Cello